ON THE ADAPTATION OF ASSURANCE FORMULÆ TO THE ARITHMOMETER OF M. THOMAS.

To the Editor of the Assurance Magazine.

SIR,—In adapting assurance formulæ to the processes that may be wrought on M. Thomas de Colmar's Arithmometer, the following expressions have occurred to me. They appear to be worthy of record, and I therefore submit them to you.

For simplicity of notation, let

$l = l_x$ or $l_{x.y}$, &c., *i.e.*, expressions into which only *lives* enter;
$d = d_x$ or $d_{x.y}$, or $d_x l_{y+\frac{1}{2}}$, &c., expressions into which *deaths* enter;
$r = (1+r)l_x$ or $(1+r)l_{x.y}$, &c., expressions into which the *rate* of interest enters;
$a = a_x$ or $a_{x.y}$, &c., *annuities* of all kinds;
$A = A_x$ or $A_{x.y}$, or $A_{\overset{1}{x.y}}$, &c., *assurances* of all kinds;

also let l_1, a_1, A_1, be the same, but *advanced one year;*

then in all cases
$$a = \frac{a_1 l_1 + l_1}{r} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$
and
$$A = \frac{A_1 l_1 + d}{r} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Of these equations, the first, in the form $a = \frac{(1+a_1)l_1}{r}$, is well known; the second is, to me at least, new; and both, for mechanical computation, are very convenient. Being symmetrical they are easy to remember.

The subjoined small table will enable any of your readers, who may be so disposed, to try these methods. I do not propose them as the fittest for working *by hand*, but I am persuaded that the days of hand work in the actuary's craft are coming to an end. The arithmometer is not an expensive machine, and its speed and certainty are invaluable.

I am, Sir,
Yours very truly,
J. HANNYNGTON.

Kingstown (near Dublin),
18th March, 1865.

Carlisle Table, 3 per Cent. Difference of Ages, 3 Years.

x.	d_x	l_x.	$l_{x+\frac{1}{2}}$.	$d_x l_{y+\frac{1}{2}}$.	$l_{x.y}$.	$(1+r)l_x$.	$(1+r)l_{x.y}$.	y.	a_x.	$A_{\overset{1}{x.y}}$.
98	3	14	12·5	79·5	420	14·42	432·60	95	2·3883366	·4829954
99	2	11	10	41	253	11·33	260·59	96	2·1308922	·5116357
100	2	9	8	32	162	9·27	166·86	97	1·6825565	·5699206
101	2	7	6	25	98	7·21	100·94	98	1·2281855	·6438465
102	2	5	4	20	55	5·15	56·65	99	0·7710435	·7270884
103	2	3	2	16	27	3·09	27·81	100	0·3236246	·7847984
104	1	1	0·5	6	7	1·03	7·21	101	0·0000000	·8321775

$$a_x = \frac{(1+a_{x+1})l_{x+1}}{(1+r)l_x}; \quad A_x = \frac{A_{x+1}l_{x+1} + d_x}{(1+r)l_x}; \quad A_{\overset{1}{x.y}} = \frac{A_{\overset{1}{(x.y)+1}} l_{(x.y)+1} + d_x l_{y+\frac{1}{2}}}{(1+r)l_{x.y}}.$$

At the highest age l_{x+1} and a_{x+1} both vanish, and at the age next below a_{x+1} vanishes; the initial value is, therefore, $\frac{l_{x+1}}{(1+r)l_x}$, where $x=103$.

At the highest age $A_{\overset{1}{(x.y)+1}}$ vanishes, and the expression becomes $\frac{d_x l_{y+\frac{1}{2}}}{(1+r)l_{x.y}}$.